

5. Given $4y^2 - 250x + 25x^2 - 16y + 541 = 0$

combine like term:

$$4y^2 - 16y + 25x^2 - 250x = -541$$

$$4(y^2 - 4y) + 25(x^2 - 10x) = -541$$

$$4(y^2 - 4y + 4) + 25(x^2 - 10x + 25) = -541 + 16 + 625$$

$$4(y-2)^2 + 25(x-5)^2 = 100$$

Divide the above eqn by 100

$$\frac{(y-2)^2}{25} + \frac{(x-5)^2}{4} = 1$$

Center $(h, k) = (5, 2)$

$a = 2, b = 5$

Vertices are $(h, k-b), (h, k+b)$

$V' = (5, -3), (5, 7)$

Focus $(h, k \pm c)$

$$c^2 = b^2 - a^2 \Rightarrow c = \sqrt{b^2 - a^2} = \sqrt{25 - 4} = \sqrt{21}$$

Focus $\Rightarrow (5, 2 - \sqrt{21}), (5, 2 + \sqrt{21})$

